## 7) EQUATIONS OF TANGENTS AND NORMALS

Formula: $y - y_1 = m(x - x_1)$

Examples:

1) Find the equation of the tangent to the circle $x^2 + y^2 - 2x - 6y + 5 = 0$ at the point $(3,\ 4)$.

$2x + 2y\frac{dy}{dx} - 2 - 6\frac{dy}{dx} = 0$ (implicit differentiation)

$2y\frac{dy}{dx} - 6\frac{dy}{dx} = 2 - 2x$ (rearrange to make $\frac{dy}{dx}$ the subject)

$\frac{dy}{dx}(2y - 6) = 2 - 2x$

$\frac{dy}{dx} = \frac{2-2x}{2y-6}$ (now substitute (3, 4) into this expression to find m)

At the point $(3,\ 4)$, the gradient is: $\frac{2-2x}{2y-6} = \frac{2-2(3)}{2(4)-6} = \frac{-4}{2} = -2.$

Therefore eqn of tangent is: $y - y_1 = m(x - x_1)$ => $y - 4 = -2(x - 3)$

=> $y = -2x + 10$

2) A curve has parametric equations $x = 3\sec\theta$, $y = 2\tan\theta$. Find the equation of the normal to the curve at the point where $\theta = \frac{\pi}{4}$.

If the equation is in parametric form, use $\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$ to find the expression for the gradient.

$y = 2\tan\theta$ $\qquad$ $x = 3\sec\theta$

$\frac{dy}{d\theta} = 2\sec^2\theta$ $\qquad$ $\frac{dx}{d\theta} = 3\sec\theta\tan\theta$

$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = 2\sec^2\theta \times \frac{1}{3\sec\theta\tan\theta} = \frac{2\sec^2\theta}{3\sec\theta\tan\theta} = \frac{2\sec\theta}{3\tan\theta}.$

At the point where $= \frac{\pi}{4}$, gradient is: $\frac{2\sec\theta}{3\tan\theta} = \frac{{}^{2}/_{\cos\theta}}{3\tan\theta} = \frac{{}^{2}/_{\cos({}^{\pi}/_{4})}}{3\tan({}^{\pi}/_{4})} = \frac{4}{3\sqrt{2}}$

The gradient of the normal is the negative reciprocal: $\frac{-3\sqrt{2}}{4}$

We need the coordinates of the point where $\theta = \frac{\pi}{4}$:

$x = 3\sec\theta = \frac{3}{\cos({}^{\pi}/_{4})} = \frac{6}{\sqrt{2}}$, $\qquad y = 2\tan\theta = 2\tan({}^{\pi}/_{4}) = 2$

The eqn of the normal is: $y - y_1 = m_n(x - x_1)$

$y - 2 = \frac{-3\sqrt{2}}{4}(x - \frac{6}{\sqrt{2}})$

$y = \frac{-3\sqrt{2}}{4}x + 6\frac{1}{2}$

Worksheet

Delta Ex 38.2 pg 379 Q1, 2